

Exercise 37

Evaluate the integral.

$$\int_0^1 (x^e + e^x) dx$$

Solution

Split up the integral into two, write the integrands as derivatives, and apply the second part of the fundamental theorem of calculus to both.

$$\begin{aligned} \int_0^1 (x^e + e^x) dx &= \int_0^1 x^e dx + \int_0^1 e^x dx \\ &= \int_0^1 \frac{d}{dx} \left(\frac{x^{e+1}}{e+1} \right) dx + \int_0^1 \frac{d}{dx} (e^x) dx \\ &= \left(\frac{x^{e+1}}{e+1} \right) \Big|_0^1 + (e^x) \Big|_0^1 \\ &= \left(\frac{1^{e+1}}{e+1} - \frac{0^{e+1}}{e+1} \right) + (e^1 - e^0) \\ &= \frac{1}{e+1} + e - 1 \\ &= \frac{1}{e+1} + \frac{(e-1)(e+1)}{e+1} \\ &= \frac{1 + (e^2 - 1)}{e+1} \\ &= \frac{e^2}{e+1} \end{aligned}$$